IA Scrpt H

Investigation of the effect of heat on the coefficient of restitution

1. Background

1.1 Introduction

Balls have always been an integral part of essentially every sport. Every sport I have enjoyed greatly always had a ball, something to bounce, throw, or catch which now raised the question to me, exactly what affects the bounce of a ball. The material and shape of is a well-known factors but I wanted to dig deeper into something else, temperature. Does playing a sport in different temperatures affect the trajectory of a ball bouncing? To sports such as tennis, basketball, soccer, and handball. Taking IB Physics I learned how the elasticity of an object affects how much energy is lost after a collision, in terms of a ball bouncing, how much of the initial height or speed is conserved after the bounce? Adding on I also know that the bonds and molecular structure of an object affect elasticity. As a result, I have chosen to investigate how temperature affects the elasticity of a ball, or in other words to quantify it, the coefficient of restitution.

1.2 Background information

As a ball is raised it gains gravitational potential energy(GPE) which can be calculated by multiplying the mass of the ball by the height and the gravitational field strength(being 9.81N/kg on Earth). When it is dropped the energy gained from gravity is converted into kinetic energy as the ball gains speed and is converted fully into kinetic energy right before hitting the ground. Elasticity is how much energy a ball conserved after a collision. However, in the real world, no rubber ball is ever fully elastic. For this IA I will be ignoring air resistance and only focus on the elasticity of the ball, or its coefficient of restitution, the amount of energy conserved during a collision.

As the ball collides with the ground it acts just like a spring meaning it will follow Hooke's Law to some extent. As the ball collides it will push the ground with a restoring force and the ground will push back with the same force but in the opposite direction according to Newton's third law(HKU Physics, 2023). However, during this collision, some energy is lost to heat(and sound and air resistance) meaning the loss in thermal energy makes it so the ball cannot go back to its original height.

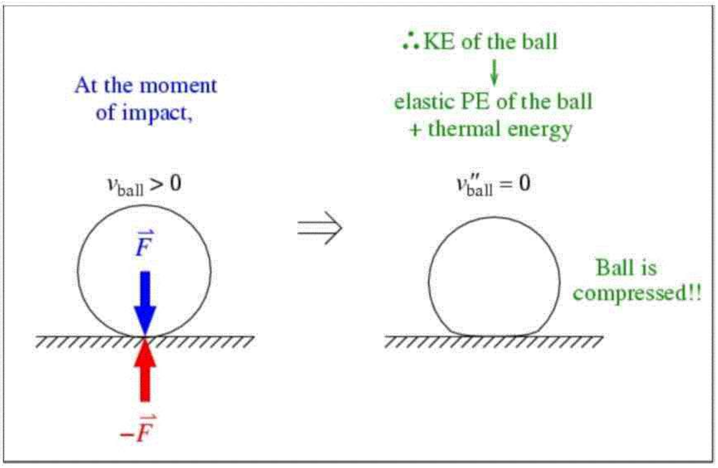


Figure 1: Diagram of how the ball is compressed in a collision(Physics.hku, 2023)

Hysteresis is the process where the ball loses energy in a collision as it compresses and extends during it. As force multiplied by distance is equal to energy, the force given to compress the ball multiplied by how much it compresses is the energy used during the compression. Using the knowledge that Young's Modulus(how stiff a material is) is more or less proportional to temperature, we can conclude that increasing the temperature increases the stiffness of the ball, making it compress less so it loses less energy. As a result, increasing the temperature should let the ball conserve more energy leading to a higher coefficient of restitution(Tamiya, 2010).

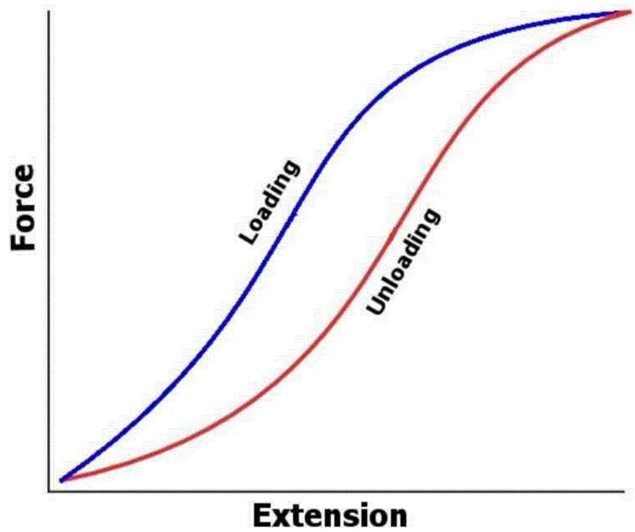


Figure 2: Example of a hysteresis diagram(ReasearchGate,2014)

The coefficient of restitution quantifies a collision's elasticity. It is the coefficient of the initial energy of the ball and the energy after the collision, showing how much energy was conserved(byjus, 2021). If the coefficient of restitution is 0, the ball conserved none of its energy and does not bounce back up at all. A coefficient of restitution of 1 means the ball conserved all of its energy and bounced back to its initial height. While the coefficient of restitution is normally measured through the initial velocity and the final velocity after a collision, in a situation where air resistance is present(so the velocity of the ball is not constant) using the height after it bounced back to is a way to measure it as well as it shows the change in GPE.

$$e = \frac{V_{Final}}{V_{Initial}} \rightarrow e = \frac{V_{Final}}{V_{Initial}} * \frac{\sqrt{\frac{1}{2}m}}{\sqrt{\frac{1}{2}m}} = \frac{\sqrt{KE_{Final}}}{\sqrt{KE_{Initial}}}$$

$$e = \frac{\sqrt{Energy_{Final}}}{\sqrt{Energy_{Initial}}} = \frac{\sqrt{GPE_{Final}}}{\sqrt{GPE_{Initial}}} = \frac{\sqrt{mg * (h_{Final})}}{\sqrt{mg * (h_{Initial})}}$$

$$\rightarrow e = \frac{\sqrt{h_{Final}}}{\sqrt{h_{Initial}}}$$

While from knowing the relationship between temperature and elasticity is proportional through Young's Modulus showing how temperature is proportional to stiffness, no equation shows the type of proportionality. In a recent study conducted in the ISB Journal of Physics, the relationship was found to be inverse exponential.

1.2 Research Question

How does the temperature (20°C, 30°C, 40°C, 50°C, 60°C) of a rubber ball affect the change in height after bouncing from a height of 1 meter(aka how it affects the coefficient of restitution)

1.3 Hypothesis

From background research and theories I hypothesize that as the temperature of the rubber ball increases the coefficient of restitution will also increase, and there will be a positive correlation. As the temperature increases the hysteresis of the ball should also increase, as a result, less energy is lost through the compression of the ball making it more rigid therefore increasing the coefficient of restitution. However, at a point, the rate of decrease of hysteresis should slow leading to the coefficient to slow its rate of increase. Adding on I also predict it will be an inverse exponential or logarithmic correlation as well.

2. Methodology

2.1 Variables

Independent Variables		The independent variable will be the **surface temperature** of my rubber ball. It will increase by increments of 10°C to obtain more significant changes in my dependent variable. I will change the surface temperature by placing the ball in a heated water tub with the targetted temperature set, I will then measure the ball with my infrared thermometer and take it out once it has reached the target temperature.I will have 5 values reaching up to 60°C as further that limit, the surface temperature reaches thermal equilibrium with the room temperature drastically faster(Newton's law of cooling) making the data gathered further from the chosen temperature. This will be measured using an infrared thermometer
Dependent Variables		My dependent variable is **the height the ball bounces back up** after being dropped from 1 meter. This will be measured from a video taken of the ball dropping off my phone and looking at the rebound height in the recording. After recording the rebound height I will then calculate the coefficient of restitution “e” by the reciprocal of the square root of that height
Controlled Variables	**Using the same ball**	The same ball is used as different balls, no matter how similar they are, will have a slight difference in elasticity
	Dropping from the same height	The ball will be dropped on the same surface, as different surfaces will affect how the ball is compressed in bounce
	Constant room temperature	The temperature of the room is more or less constant, which will affect how the ball changes temperature while a trial is being conducted
	Same ground for a bounce	I will be experimenting the same place for every trial, this is to ensure that the material and shape of the ground

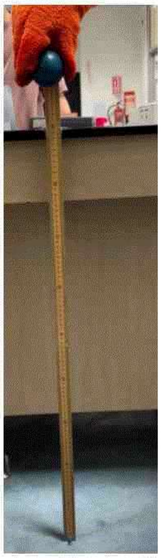
		remains constant as that can change the ball's motion after bouncing
	Dropped with no initial velocity	I will be dropping the ball with no force added to it at the start, this will make sure there is no initial velocity at the start of the drop. All forces acting on it will be gravity.
	Camera position	The iPhone camera will be placed close to the ball drop and be at a 90° angle to the ground. This will eliminate any distortion from the angle of the camera lens affecting the recorded rebound height.

2.2 Equipment and Apparatus

- Rubber Ball
- Water Bath(± 1 x 10^{-2} °C)
- 1 Meter Stick (± 5 x 10^{-4}m)
- Laser Thermometer (± 1°C)
- iPhone Camera

2.3 Procedure

1. Place the rubber ball into the water bath set 5-10°C higher than the wanted temperature(to ensure it stays around the wanted temperature during the trial as it cools to room temperature) and wait until the ball reaches the set temperature
2. Take the ball out of the bath and use the infrared thermometer to measure its temperature(to check if it is at the desired temperature or a bit above)
3. Position the bottom of the ball at the top of the meter stick(at a height of 1 meter to the ground)
4. Start recording
5. Drop the ball with no initial velocity
6. After the ball bounces, catch it after it drops below rebound height
7. Repeat the process for 5 trials, do not wait before each trial do it as soon as you catch it
8. Stop recording
9. Repeat for 5 more IV values
10. Check the height of the bottom of the ball when the ball stops moving upward



2.4 Ethical, Safety, and Environmental Concerns

There are no ethical or environmental concerns, with this experimental setup. The water in the water bath can be reused for every temperature leading to no extra water consumption, this is the best option for preventing any environmental issues with using water. However there is a safety concern that comes with this, gloves are essential. As it is required to take out the ball from the water bath and catch it, as the temperature is increasing it is necessary to wear thick gloves. Especially at higher temperatures as they can cause serious burns when in contact.

2.5 Qualitative Data

As the ball dropped to the floor there was a slight but constant rotation. This does not a significant difference to my data however when there was a slight rotation towards the camera the ball bounced forward a little which in hindsight would not affect the actual rebound height, but the rebound height the camera captures. This is because the ball would be closer to the camera.

2.6 Quantitative Data

Raw Data

	Bounce Height After Dropping from 1 meter(m) ±0.001m				
Tempreture(°C) ±1°C	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
20	0.62	0.63	0.62	0.62	0.63
30	0.69	0.67	0.7	0.71	0.7

40	0.7	0.7	0.69	0.7	0.7
50	0.71	0.7	0.71	0.71	0.71
60	0.73	0.76	0.75	0.76	0.75

Processed Data

$$coeffecient\ of\ restitution\ =\ \sqrt{\frac{average\ rebound\ height}{1}}$$

Temperature(°C) ±1°C	**Average**	**Average Coefficient of Restitution ±0.002**
20	0.624	**0.789**
30	0.694	**0.833**
40	0.698	**0.835**
50	0.708	**0.841**
60	0.75	**0.866**

2.7 Sample Calculations

<table>
<tr>
<td>Calculating average height for each trial

Average Height for trial 1 = $\frac{0.62 + 0.63 + 0.62 + 0.62 + 0.63}{5}$
= 0.624m</td>
<td rowspan="2">Calculating uncertainty for Coefficient of Restitution for trial 3

Uncertainty for ruler → 5 x 10^{-4}m

Absolute Uncertainty for height = 1 x 10^{-3}m (measuring from two points so 5 x 10^{-4}m * 2)

Absolute Uncertainty of Average Height = $\frac{0.001}{2}$ = 0.0005

COF formula = $\sqrt{\frac{average\ rebound\ height}{1}}$

Relative Uncertainty of COF =$\left|0.5\ *\ \frac{0.0025}{0.698}\right|$ = 0.0018

Absolute Uncertainty = 0.835 * 0.0007 * 2 = 0.002</td>
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<tr>
<td>Calculating coefficient of restitution of trial 2

COF_{Trial_2} = $\sqrt{0.694}$ = 0.833</td>
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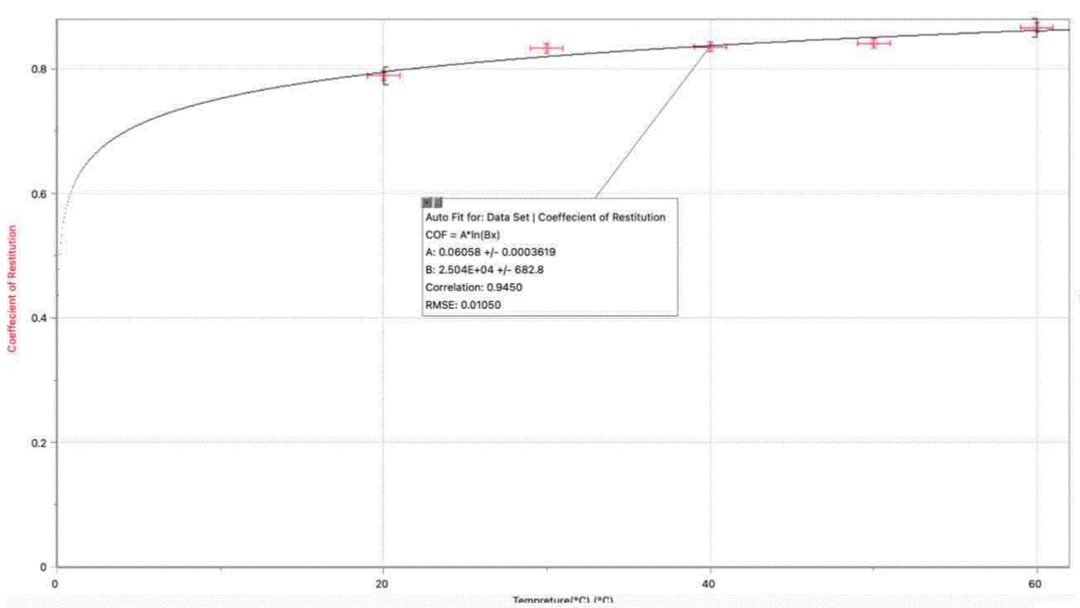
	As the other values are very small, the absolute uncertainty for each trial rounds to 0.001 as well

Graph 1: Processed Graph of How Temperature Affects Average Coefficient of Restitution



3. Analysis and Evaluation

3.1 Conclusion

From Graph 1 my initial hypothesis was correct, there is a positive correlation between the temperature of a ball and its coefficient of restitution. Additionally in my hypothesis, there is an inverse exponential(logarithmic) relationship present as well. The equation for the relationship is 0.06ln(25040x)= y, x is the temperature of the ball in °C, and y is the coefficient of restitution. This means the coefficient increases as the temperature increases, but the rate of change decreases as the temperature approaches high values. After a slight jump from 20°C to 30°C, the rate of change plateaus as the coefficient increases but increases very slowly, from 30°C to 60°C there is only a 0.033 increase.

This relationship is what I expected from my research. The temperature of the rubber ball increased, and the hysteresis increased as well but once the ball reached higher temperatures the hysteresis did decrease but it did not fully disappear at those temperatures(Tamiya, 2010). This proves that as the temperature increases and the hysteresis increases as well, there comes a point where the hysteresis increases slower as it cannot go to 0, leading the coefficient of restitution to decrease slower as well, leading to a plateauing graph.

3.2 Sufficiency

There were 5 variations for the temperature with each variation having 5 trials. There were not any outliers in the experiment but in the 2nd trial(30°C) there was a larger range of heights, a range of 0.4 which is higher than all the other trials. To construct the curve of best fit, 5 data points were enough to ensure higher sufficiency more variations(or shorter increments between them) and a greater number of trials may have been needed.

3.3 Reliability and Accuracy

There isn't a theoretical value for this experiment just a relationship, as a result, it is hard to quantify accuracy, however from looking at the errors of my equipment and the reliability can give a good picture.

The correlation coefficient is 0.945, which shows a very strong correlation between the two variables. It is strong enough to ensure relatively high reliability to conclude that there is a positive relationship between the temperature of a ball and its coefficient of restitution.

Adding there was low systemic error as all the curve passed through all the data points but the uncertainty propagated for the coefficient of restitution was very low, each data point having an absolute uncertainty for the COF of about 0.001. As the equation has the height square rooted it decreased the uncertainty, as a result, the curve did not pass through the error bar for one data point(at 30°C). However, it passes through every other point, and the gap between the error bar at 30°C and the curve is miniscule. As a result, this small issue is not enough to completely oppose the claim that temperature and the coefficient of restitution have a positive correlation.

4. Strengths, Weaknesses, and Improvements

4.1 Strengths

One major strength of this experiment was the usage of the same ball for every trial/variation. This is vital as different balls, even if they are of the same material, based on their size and shape can behave differently when bounced. So using the same ball ensured that all the data points collected were valid for that object.

One major strength of this experiment is the way of conducting all the trials of each IV simultaneously. After the ball reached its rebound height I picked it up and dropped it for the second trial subsequently with no pause, this made sure the least amount of heat loss occurred. Each variation had its 5 trials conducted in under 10 seconds and I checked the temperature of the ball after that it fluctuated in a 2-3°C range by the last trial.

Another strength is the usage of a relatively small ball and a short drop height to minimize air resistance. Air resistance makes the ball lose energy as it drops and bounces up so reducing it was a necessity. I used a relatively small ball so it would have a smaller cross-sectional area, meaning less air would collide with it as it fell. Adding on a sphere is the optimal shape of an object to reduce drag or air resistance. Finally, I used only a meter drop height, what this does it is shortens the time in which the ball interacts with the air, since the longer the ball travels the more air resistance increases and it approaches its terminal velocity.

Weakness	**Explanation of Weakness**	**Improvements to be made**
Heat loss from the surrounding environment	While the trials were done in a very fast manner there was still some heat loss from the surroundings. As the ball was being dropped and exposed to room temperature, heat loss was noticeable. The first IV, of 20 degrees was colder than room temperature(24°C) and the same things happened as the ball and the air around it tried to reach thermal equilibrium	- Using material that also is resistant to heat loss - Conducting the drop in a temperature controlled tunnel
Distortions of the iPhone camera	While the camera was placed at 90° to the drop, there was prone to be some distortion. First of all, if the ball were to bounce forward a little the height would be distorted, and as this was shot at 30 frames per second, the rebound height may have been higher which was not captured.	- Use a higher frame rate camera - Have a second camera on the side to show any forward or backward movement
Ground collision impacted by wet surface	As the ball had to be dropped as soon as it got out of the water bath for the least heat loss, a problem was sustained of the ball being wet as it was being tested. This is problematic as a wet surface changes the properties of a material when colliding which will change the motion of the ball after.	- Use a different material ball that dries quicker or use a cloth to dry it
Ignoring air resistance	While I did use a short distance and a ball with a relatively small cross-sectional area, the air resistance was decreased but it was still present. This means that the coefficient of restitution	- Use a smaller ball with a smaller cross-sectional area - Use a vacuum tube big enough to

	be higher for all trials as some energy was lost to colliding with air molecules.	fit a rubber ball
Unequal distribution of heat among the ball's surface	When heating the ball in the water bath, the ball floated in the water. This is an issue as it means the part of the ball in the water will experience more heat than the one outside. As a result, one area of the ball will have a higher temperature than the other.	- Roll the ball around in the water bath and keep it for a longer time inside - This will make sure all parts of the ball get the same heat and it is heated long enough to have more equal distribution. I will them check this by using a thermometer to check multiple sides of the ball

5. Extension

I suggest that an extension to this experiment is to conduct it in a vacuum tube and use a hollow wooden ball, as wood does not tend to bounce making it hollow will make it more elastic/bouncy. Air resistance decreases the energy of an object in motion affecting the results for the COF, therefore dropping the ball in a vacuum chamber eliminated the effect of it on the energy of the ball after bouncing. Since heat loss from any material is unavoidable, using a material with a high specific heat capacity reduces the effect as it takes more energy to change the temperature. Rubber has a specific heat capacity of about 2005 J kg^{-1} K^{-1} while oak wood has around 2380(Evans). As a result of its higher specific heat capacity, using a wooden ball would be better as it would have a smaller fluctuation in temperature while conducting trials.

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